IMO third selection test – Piteşti, May $24^{\text{th}}$ 2003

**7.**

Find itnegers $a, b, m, n$ with $m > n > 1$ such that the polynomial $g = X^n + aX + b$ divides $f = X^m + aX + b$.

**8.**

Two circles $\mathcal{C}_1$ şi $\mathcal{C}_2$ with radii $r_1$ and $r_2$, $r_1 < r_2$ are externaly tangent. The line $t_1$ is exeternaly tangent to the circles $\mathcal{C}_1$ and $\mathcal{C}_2$ at points $A$ and $D$ respectively. The parallel line $t_2$ to the line $t_1$ is tangent to the circle $\mathcal{C}_1$ and intersects the circle $\mathcal{C}_2$ at points $E$ and $F$. The line $t_3$ through $D$ intersect the line $t_2$ and the circle $\mathcal{C}_2$ at points $B$ and $C$ respectively, different than $E$ or $F$.

Prove that the circumcircle of the triangle $ABC$ is tangent to the line $t_1$.

**9.**

In $n \times n$ array $(n \geq 2)$ the real positive numbers $a_1, a_2, \dots a_{n^2}$ are placed. Suppose that

$$\sum_{k=1}^{n^2} a_k = n^3.$$

Prove that one can find four numbers which are the vertices of a square (with a side parallel to the horizontal) having the sum greater than $3n$.